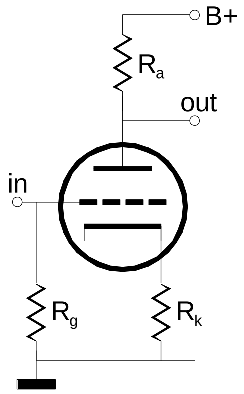


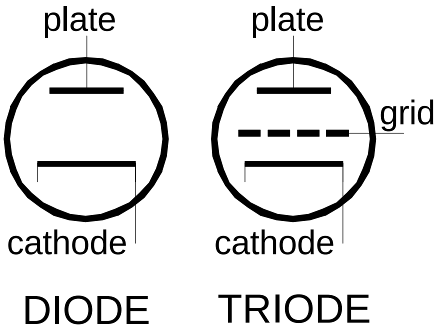
The Grounded-Cathode Amplifier

Determining cathode and plate resistor values

The grounded-cathode amplifier is approaching its centennial (2007) and it remains the building block of most tube audio equipment. It is simplicity itself, with little more than a few resistors and a triode; yet many are ignorant of its inner workings.

Mechanical engineers must know the basics of their practice — lever, wedge, wheel and axle, pulley, and screw — as these five simple machines form the basis of all other more complex machines. Likewise, vacuum tube circuit designers must know the inner workings of the grounded-cathode amplifier, the cathode follower, and the grounded-grid amplifier, as these three basic circuits form the basis for nearly all complex tube circuits.

If for some reason we could understand only one of the three basic tube circuits, which circuit should we chose? Undoubtedly, the best choice would be the grounded-cathode amplifier, as it finds the widest use and it embodies the inner workings of a vacuum tube nicely. Yet many do not know how this simple tube circuit works, neither knowing how to select a suitable cathode resistor to set the circuit's idle current, nor how to chose a useful plate resistor (aka anode resistor) value to allow the greatest swing of output voltage. This article offers a simple explanation of the triode and explains how to determine these two resistor values.

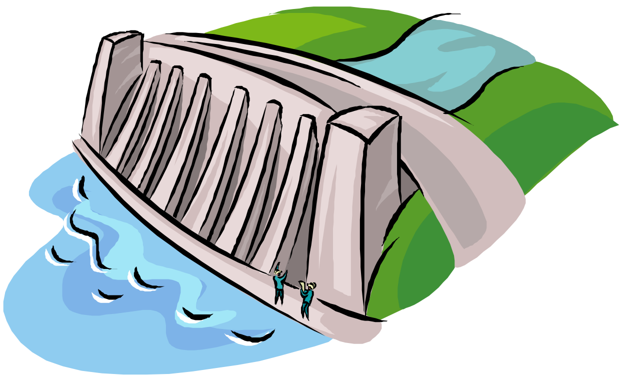


A Little History and Theory

The triode descends from the tube diode (tube rectifier) and as a consequence it shares several features with the diode. For example, it can only conduct current in one direction: from its cathode to its plate (or from cathode to grid when the grid is more positive than the cathode). In addition, it offers some resistance to the flow of that current, which means that like the tube diode, the more current flowing through it, the greater the voltage drop across it. (In fact, if we tie the grid to either the cathode or the plate or if we leave it unconnected, the triode is reduced to a diode.) This resistance is referred to as the triode's r_p or plate resistance. (In the old days, r_p was reserved for references to the triode's AC impedance; and R_p, the triode's DC resistance. Sadly, today, r_p covers both the DC and AC aspects of a triode's behavior.)

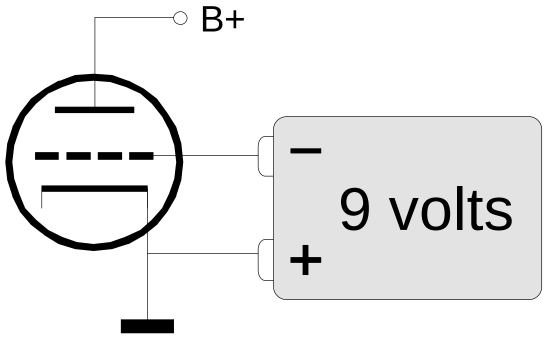
By allowing the easy control of the current flowing through the triode, the grid makes the triode a useful electronic device. The poor relation, the diode, lacks this feature, as the only way to control the flow current through the vacuum diode—other than to reverse the polarity of the applied voltage—is to vary the voltage between its plate and cathode. In fact, if the diode did not hold the property of unidirectional current conduction, it would be of no more use than a slow-turn resistor, with a relatively short life expectancy and poor linearity. In contrast, by simply varying the voltage on its grid, the triode's current can be completely stopped — or increased up to complete saturation.

In other words, the grid is like the valve in a water faucet; so much like it, in fact, that the vacuum tube is often referred to as a valve. Because turning a water faucet on and off requires so little effort, we cannot see the splendor of the stunt. So, instead, imagine a large dam and its water valve; it is a huge reservoir of potential energy, yet one individual can turn the handle (the valve) that releases a dangerous flood. The amount of work required to turn the handle is nothing compared to the water's release and once the effort has been expended opening the valve, it remains open without any further expenditure of work on our part.



In much the same way as the dam's flood valve, the triode's grid presents an extremely light load, considering the large resulting effect. Unlike a transistor's low-impedance base, the grid's input offers so great an impedance that one of the triode's last jobs, before being replaced by solid-state devices, was as the input device on voltage meters and oscilloscopes, where its high-input impedance prevented excessive interaction with what was being measured and its ability to survive high voltage mishaps added reliably to the test instruments.

Here is an example that illustrates how little current a grid conducts when it is substantially more negative than the cathode. If a 9-volt battery is connected to a triode so that its negative terminal attaches to the grid and its positive terminal attaches to its cathode, while the cathode is grounded and its plate is connected to a high voltage power supply, the battery's life expectancy would probably prove no shorter than that of its brother still shrink-wrapped at the store. Yet, the battery is an essential part of the circuit.



Like the amount of work required to release the dam's flow, most of the work required to adjust the triode's current came at the beginning. Charging a capacitor requires energy, just as filling a swimming pool requires water and pressure. In this example, this one-time connection results in an insignificant amount of work. Make and break this same connection a million times per second and the battery will be taxed and depleted, as now the recharging of the capacitance represents much more work. (Imagine what your hands would look like after turning the water on and off a million times per second.) Not all tubes would require the same amount of work, however, as each type has a differing amount of capacitance.

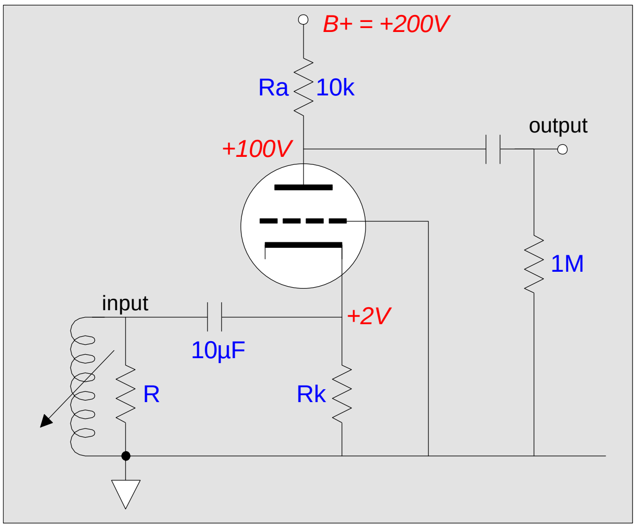
In the battery-biased circuit shown above, the circuit without a plate resistor, the varying amounts of capacitance would be the only issue, but in an amplifier circuit, wherein the plate works into a load (such as a resistor or a transformer), the amplification realized by the triode becomes crucial to determining the amount of work required to recharge the capacitance.

This is so because the amplification performed on the input signal also acts on the capacitance between the grid and the plate, amplifying it; thus making it much more of a burden. This is, in a nutshell, the dreaded Miller effect: the multiplying of the grid-to-plate capacitance by the gain of the triode.

Cathode

So far we have covered only one of the triode's three controlling elements (the grid). The cathode can also be used to control the flow of current through the triode.

In fact, the cathode is slightly more effective at controlling the current flow than the grid. But unlike the grid, the cathode presents a low-impedance input and thus requires more effort to move its voltage up or down. (When the cathode is used as the control element, i.e. as the input, the triode is being used in the grounded-grid topology.)



Grounded Grid Amplifier

The plate can also control the flow of current, as increasing the cathode-to-plate voltage increases the flow of current. The plate, however, is not as effective as the grid or the cathode in controlling conduction. Where the grid might need to see a 1-volt change in voltage to incur a 10-mA increase in current flow, the plate might require a 100-volt change to yield the same 10-mA increase...which brings us to mu.

mu, G_m, and r_p

The *ratio* of the plate's effectiveness over the grid's effectiveness in controlling current flow from cathode to plate defines the *mu* or *amplification factor* or μ of a triode. And the measure of any controlling element's ability to vary current conduction in response to a change in its voltage goes by the name of *transconductance*. Each of the triode's three elements displays its own amount of transconductance. The most commonly specified transconductance is that of the grid, which often is labeled G_m or *mutual conductance* and noted in micro-siemens, the siemens (S) being the unit of conductance, the inverse of resistance. The plate's transconductance equals $1/r_p$; the grid's, mu/r_p; and the cathode's, $(mu + 1)/r_p$.

A quick review: the triode can only conduct current from its cathode to its plate (and, in some cases, to its grid); the triode offers resistance to the flow of current that results in a voltage drop across the triode, much like the voltage drop across a resistor; and both the grid and the cathode are much more effective than the plate in controlling the flow of current through the triode. Given this short explanation of the triode's workings, we can move on to how to set a triode's idle current with a cathode resistor.

Cathode Bias

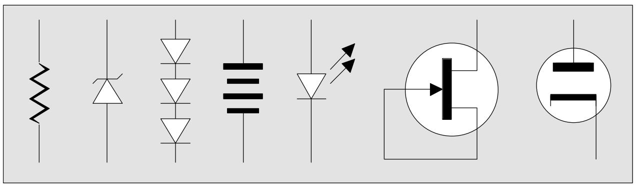
In the absence of a cathode resistor, with both the grid and the cathode seeing the same voltage, the triode's r_p offers the only controlling opposition to the flow of current through the triode. This configuration can result in a great deal of current, as V_p (the cathode-to-plate voltage) divided by the triode's r_p, roughly equals the amount of current. For example, a triode with an rp of 2k under a V_p of 100 volts will draw 50 mA of current, which against the 100 volts equals 5 watts of heat dissipation by the triode. Doubling the B+ voltage would also double the idle current and thus quadruple the dissipation, as both the current and the voltage have doubled.

How can we alter this circuit to decrease the idle current, while retaining the same V_p? Two avenues present themselves. The first technique is to make the grid more negative than the cathode by connecting the grid to a negative power supply, such as the battery from the previous example. The advantage of a negative power supply is that a simple potentiometer can be used to adjust the idle current. How negative should this power supply be?

Consider this: because the grid is mu times more effective than the plate in controlling current, making the grid voltage equal the B+ voltage divided by the mu and made negative, should turn the triode completely off. Expressed mathematically:

$$I_q = 0 \text{ when } V_{gk} = -V_b/mu.$$

(If triodes were perfect, then "should" would have been replaced with "will." Unfortunately, as good as triodes are, they are not perfect, particularly at the bottom of their conduction near cutoff. Still, this is a handy equation to memorize.) Thus, the negative power supply need only be 1/mu times as large as the B+ power supply to ensure a wide range of current control.



Cathode-Biased Voltage Dropping Devices

The second technique to decrease the idle current is to make the cathode more positive than the grid by inserting a voltage drop between the cathode and ground. We could use a battery, an LED, a zener, a solid-state diode, vacuum tube diode, or even a second power supply, but a resistor is the most common choice. The larger the cathode resistor's value, the greater the effective negative bias voltage.

So could a cathode resistor's value ever be so great as to turn off a triode? No, it never could be so large, as some current must flow to define a voltage drop across the resistor, so the triode could not be turned off.

At first glance, the resistor would seem the poorest choice as, unlike most of the other devices, it does not present a fixed voltage. But in fact, the resistor's current-dependent voltage drop is just what is needed to ensure the most consistently stable idle current.

An individual triode differs from other triodes of the same type and, over time, it even differs from itself. In other words, the fixed voltage relationship between the cathode and grid that worked perfectly with one tube may not work so well with another triode of the same type or even with the same triode two years from now.

By using a cathode resistor, however, we introduce a feedback mechanism into the circuit. As the triode increases in conduction, a larger voltage drop develops across the cathode resistor, which in turn makes the grid less positive relative to the cathode, reducing the triode's conduction. Conversely, as the triode decreases in conduction, a smaller voltage drop develops across the cathode resistor, which in turn makes the grid more positive relative to the cathode, increasing the triode's conduction. This is feedback of a truly negative sort. (In contrast to everyday speech, for most of electronic practice, *negative* feedback is desired and *positive* feedback feared, as positive feedback can lead to dangerous oscillations.) The negative feedback results in a welcome increase in consistency.

Okay, cathode bias seems like a good idea, but how do we chose the right value for the cathode resistor? Five approaches present themselves before the tube circuit practitioner.

First, *look up the value* (in a book or magazine — or ask a friend). While this is the most popular method, it is the least universally applicable, as many triodes are never mentioned and of those that are, only a few bias points are given.

Second, *work out the value* from a single simple formula. This does have a universal application, but it relies on an idealized model of the triode.

Third, *extrapolate the value* from inspecting the triode's plate curves. This method has the advantage of using the actual triode's characteristics, but requires a good eye and some skill; worse still, not all triodes find their corresponding plate curves in print.

Fourth, *track down the value* by actual experimentation. This requires both some physical work and some expense, as the triodes and resistors must be purchased and several triodes should be tested to preclude having a gassy or a weak triode throw off the results.

And fifth, *use a SPICE program to determine the value*. This is actually something of a combination of the first four approaches. The limitation here is that the result is only as good as the SPICE models of the triode used and most of the models I have seen are just okay — and there are not that many of them out there (far fewer in number than the number of published plate curves) — but the models are steadily improving and growing in number.

For most tube circuit designers the methods that work best are the second (working out mathematically) or third (inspecting the plate curves) because these methods quickly give reasonably accurate results. The formula for determining the cathode resistor's value (in the absence of a plate resistor) is a simple one:

$$R_k = \frac{(V_b / I_q) - r_p}{mu + 1}$$

where, R_k equals the cathode resistor; V_b, the B+ voltage; I_q, the desired idle current; r_p, the plate resistance; and mu, the triode's amplification factor. (If the result is a negative number, then the desired idle current requires a lower r_p triode.) For example, a 6SN7 (r_p = 7700, mu = 20) with B+ voltage of 250 volts and a desired idle current of 9 mA will need a cathode resistor of 956 ohms, which against the 9 mA idle current equals 8.6 volts and is close to the tube manual's 8 volts. In fact, in actual practice, this value would probably be close enough, or at least close enough to allow easy fine tuning. (Adding a plate resistor to the circuit will also serve to promote consistency in idle current and its value comes from the voltage drop divided by I_q.)

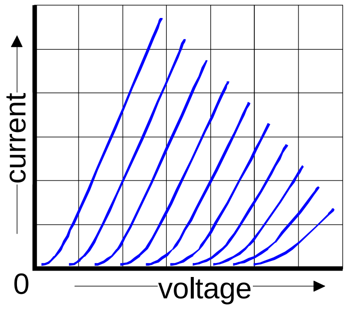
Neophytes are often horrified by 5% differences between a circuit's actual functioning and the calculated results; whereas a seasoned circuit designer would revel in such exactitude.

While vacuum tubes are much more consistent than other discrete active devices such as transistors, MOSFETs, and FETs, if you want tight tolerances, then look to passive devices, as 0.01% tolerances only exist in passive components such as resistors and capacitors.

Still, why so large a difference? Part of the answer is found in the formula's assumption of a perfect triode. Another part is found in the tube manual's imprecision in stating the mu and the rp of the 6SN7 which, in this case, forms the larger part of the error. A careful inspection of the plate curves reveals that at 250 volts and 9 mA, the mu is closer to 21 and the r_p is closer to 8k or 9k. If we recalculate the cathode resistor's value using these revised values, then much of the difference disappears between formula and plate curves.

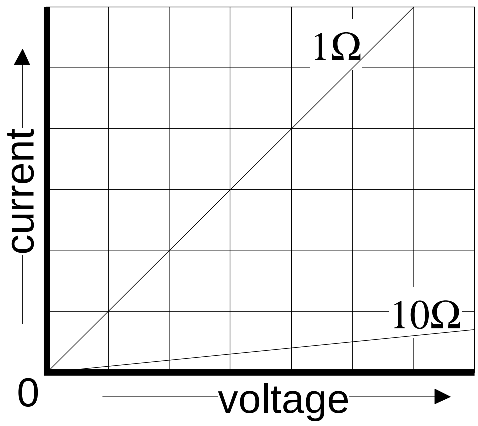
Aren't a triode's mu and rp fixed and immutable? No. Unlike a tube's dimensions and mass, its r_p, mu, and G_m vary, depending on plate voltage and plate current. The least varying characteristic is its mu; which is ironic, as it is the least *real* of the three specifiers, defining only the relation between r_p and G_m, not any actual physical aspect of the triode's design. (Naming the ratio between a man's height and the circumference of his waist "phi" might be useful for predicting heart attacks, but phi is not *real* in the same way that his eyes or kidneys are real.)

Often, most of the blame for inaccurate results lies with the difference in cathode-to-plate voltage that the tube manual specifies versus the voltage the formula assumes. The tube manual assumes that fixed bias will be used, thus retaining the full B+ across the tube, whereas the formula accounts for the voltage lost across the cathode resistor.



Extrapolation from Plate Curves

The detective has fingerprints; the fortune teller, a crystal ball; and the tube amplifier designer, plate curves. Each promises to reveal secrets. *Who did it? What will the future bring? Will it work?* Like the fingerprint, the tube's plate curves individualize the tube, which helps us decide which tube should be used. Like the crystal ball, a set of plate curves helps us predict how an amplifier will function, allowing us quickly to find the required grid voltage to set a given bias point and to determine maximum voltage swings. But before we tackle the vacuum tube's plate curves, we need to cover something much easier: simple resistances and straight lines.



Simple Resistances

The movement from the graph's left to its right defines increasing voltage; from its bottom to its top, increasing current. This arrangement allows us graphically to display Ohm's Law:

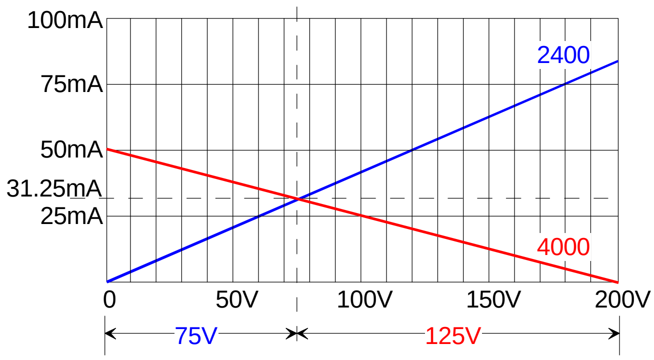
Resistance = Voltage / Current.

For example, 1 ohm equals 1 volt divided by 1 amp and 10 ohms equals 10 volts divided by 1 amp. Graphing these resistances is easy.

We know we must start at the bottom left corner where voltage and current equals zero, as no voltage across a resistance means no current. The next point for the 1-ohm resistance would be one unit (1 volt) to the right and one unit up (1 amp); and for the 10-ohm resistance, it would be ten units to the right and one unit up. Remember, the lines continue infinitely, as the relationship between voltage and current defined by the line is constant. So if we follow the 1-ohm line out to 100 kV, we know that the current will be 100 kA. Of course, no 1-ohm resistor on earth could withstand the test; but the issue here is pure resistances and not necessarily actual physical resistors.

The plotting of these two resistances reveals a key feature of graphing resistances: the lower the resistance, the steeper the line defined; and the higher the resistance, the flatter the line defined. This will prove valuable when working with tubes, as this principle allows quick inspection of plate resistances. Given that both graphs share the same X-Y scales, the steeper set of plate curves belongs to the vacuum tube with the lower plate resistance.

As the Ohm's Law formula is so simple, plotting different resistance values is usually of little value. But if we place two resistances in series and set a B+ voltage, then we can graphically see the voltage division between resistances.



Two Resistances In Series

When two resistances are in series (no resistance equaling zero) and are placed across a fixed voltage, one resistance will steal voltage from the other.

If the resistances equal each other, then each will share half the available voltage. If one resistance is twice the value of the other, then it will grab 2/3 of the available voltage.

Graphically plotting the voltage ratio is easy enough. We start with the resistor that connects to ground, i.e. 0 volts. Fix the first point at 0 volts and 0 current and then place the second point at the intersection of the maximum voltage and the maximum current that this resistor would see at that voltage based on the formula:

I = V/R.

Now to plot the resistor that connects to the B+ voltage, we start at the other end of the graph at the maximum B+ voltage and zero current. (This makes sense, because if the value of the resistance were 0 ohms, then the current would be 0 mA.) Moving to the extreme left, we find the value of current this resistor would draw, if it experienced the full B+ voltage; once again, this equals I = V/R. The result of our efforts is two lines crossing each other. The intersection of the two lines defines the voltage at the connection of the two resistances and it defines the maximum current the circuit will see with the given B+ voltage.

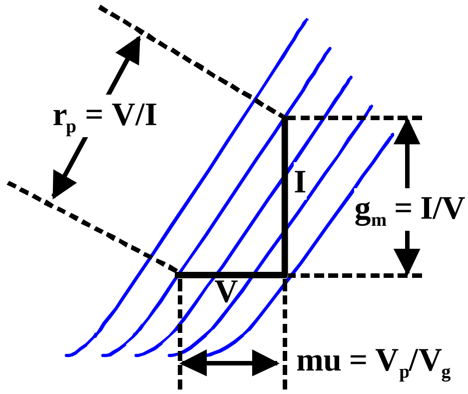
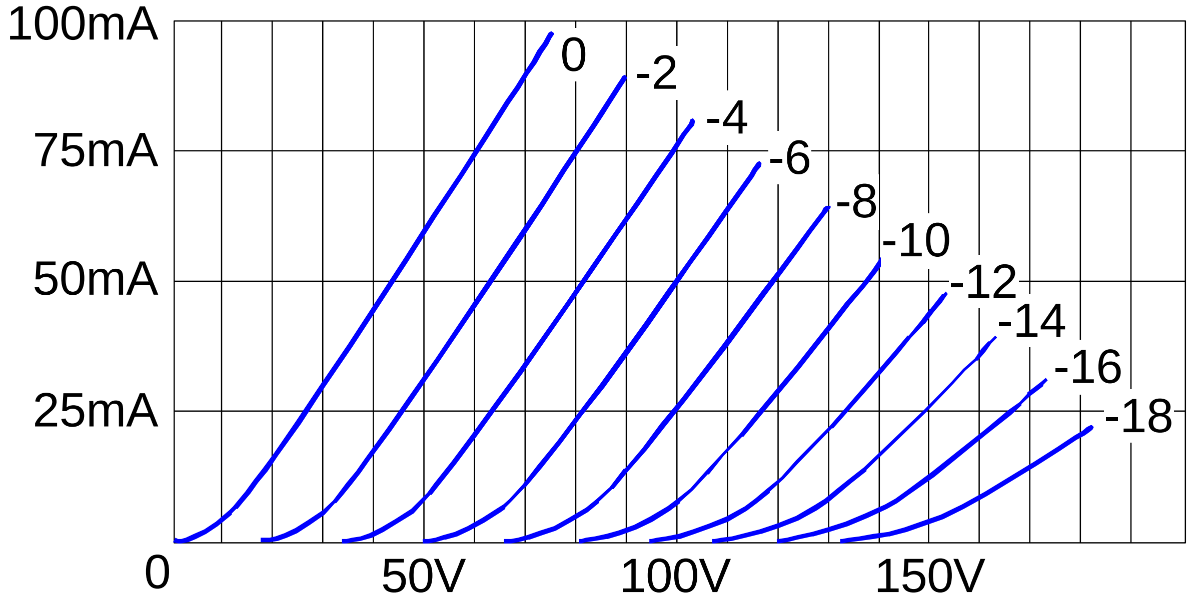


Plate Curves

Finally, we get to the triode's curving grid lines. Each line represents the triode's current conduction plotted across an increasing voltage, while a constant grid voltage is applied. The reason this set of curves goes by named "plate curves" is that each point on a curve reveals the triode's plate current at that point. For example, in the graph below, we see that with a grid voltage of -6 volts and a cathode-to-plate voltage of 100 volts, the triode conducts 50 mA of current. So does the rp equal 2k, as 100 / 0.05 = 2000? The answer is no, as once the current climbs over 15 mA, the -6-volt grid line reflects a slope closer to 714 ohms. Yet a 714-ohm resistor would draw 140 mA at 100 volts, not just 50 mA.



Why? What has happened is that product of this triode's mu (about 10) against its grid's -6 volts (-60 volts) has been added to the 100 volts to yield 40 volts as the effective plate voltage, which when divided by 714 ohms results in a current draw of 56 mA. (The grid line curves enough to make up the 6 mA difference.) Had we focused on the 0 volt grid line instead, then the plate resistance would have been much more accurately directly deduced from dividing the plate voltage by the plate current along any point on the gridline. (Once again, the lines curve enough to introduce a small error.)

Determining the value of the required cathode resistor to set the idle to 50 mA at 100 volts seems obvious enough: just divide 6 volts by 50 mA; unfortunately, this method gives an inaccurately high resistor value, as the full 100 volts will no longer be available to the triode once the cathode resistor displaces 6 volts of the B+ voltage. So the set of curves should be mentally shifted -6 volts to the right. For most beginners this is asking too much. The quick workaround is to take the apparent bias voltage (-6 volts, in this example) and divide it by the reciprocal of the triode's mu (10, in this example) and then subtract this value from the apparent negative bias voltage, which when simplified and expressed as a formula becomes:

$$V_{bias}' = V_{bias}(1 - 1/mu)$$

$$R_k = V_{bias}' / I_q$$

In this case, the -6 volts becomes 5.4 volts, which divided by 50 mA equals 108 ohms, the correct value. Obviously, the higher the mu, the smaller the adjustment becomes; consequently, with a high-mu tube like the 12AX7, often no workaround is sought, but it is essential with low mu triodes, such as the 2A3, 6AS7, 6BX7, 6C33, 300B and 845.

The previous examples relied on circuits without plate resistors. Factoring in the role played by the plate resistor only slightly increases the complexity of the task. The formula for determining the cathode resistor's value must be modified:

$$R_k = \frac{(V_b / I_q) - r_p - R_a}{mu + 1}$$

where, once again, a resulting negative value betrays that the specified idle current cannot be met under the cathode bias configuration and a lower-r_p triode or a lower-valued plate resistor is required.

Graphing the added plate resistor is easy enough, as it relies on the method used to plot the two resistors in series. We start at the B+ voltage, which in this example we will set at 200 volts, and place our first point.

Then we find the intersection with the leftmost portion of the graph by using Ohm's law:

$$I = V/R.$$

Given a plate resistor of 2k, the intersection occurs at 100 mA. The last step is to draw a line connecting both points. This line defines the range of possible idle currents and plate voltages with this plate resistor in place.

For example, if we pick 25 mA as a suitable idle current, then the plate resistor will see 50 volts across its leads and the triode will see 150 volts minus the cathode bias voltage (about 12.5 volts).

Working Backwards

Sometimes we are presented with an existing circuit that lists the values of the plate resistor and cathode resistor, but not the operating voltages or idle current. Fortunately, we can work backwards from the resistor values to the operating points (if the B+ voltage is specified). The mathematical approach involves rewriting the previously given formula:

$$I_q = \frac{V_b}{(R_k[mu + 1]) + r_p + R_a)}$$

and

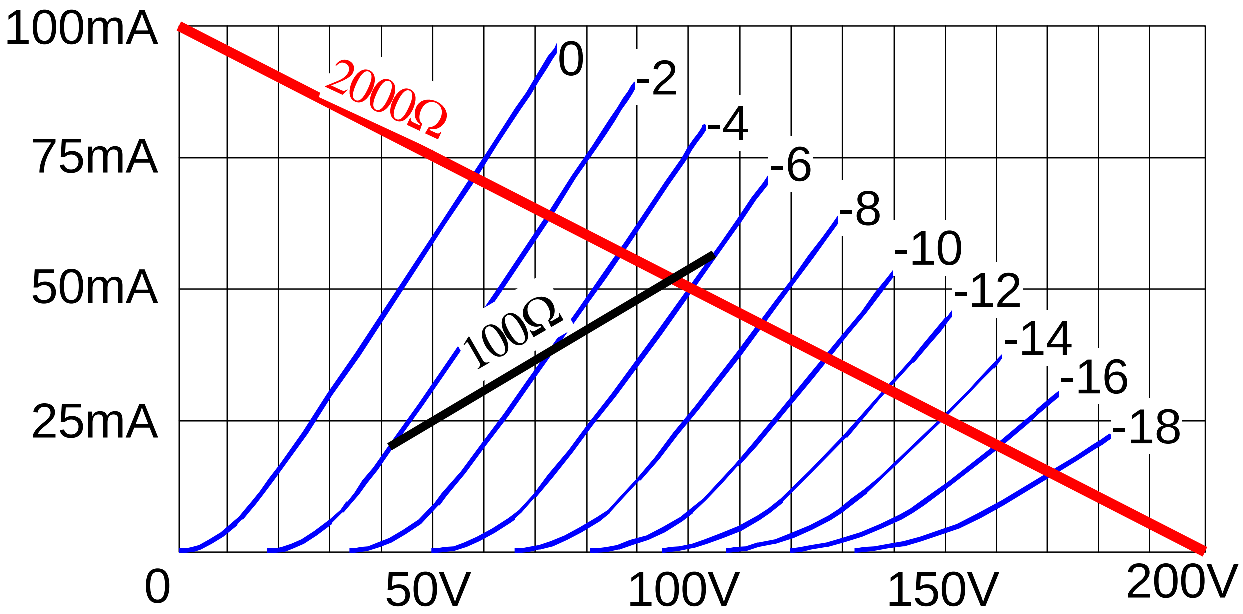
$$V_p = V_b - I_q(R_k + R_a)$$

and

$$V_{gk} = I_q R_k$$

While the results depend on the triode's mu and rp having been accurately defined from some point close to the actual operating point, the results are usually close enough to get a good idea of what is going on in the circuit.

Inspecting the plate curves promises greater precision, but requires a good eye. The first step is to plot the plate resistor as we did before. The second step is to plot the cathode resistor line. Be sure to resist the temptation to plot its line in the same fashion as we did in the resistor-only examples, as the cathode resistor does not see the voltage marked along the x-axis! The voltage the cathode resistor sees is the *potential* between ground and the cathode. Consequently, we must use the gridlines to provide the voltage needed in the formula: current = voltage / resistance. But as the gridlines curve, the cathode resistor line must also slightly curve.



The first step is to find the intersecting points on the gridlines where the corresponding current equals the current draw from the cathode resistor at the gridline's voltage (in absolute terms). This sounds complicated, but isn't. For example, in the graph above we see the plate resistor's line plotted from one corner to the other and we see the cathode resistor's line stretching from the intersection with the -2 and -6 gridlines. At these two points (20 mA and 60 mA) the 100-ohm resistor will find 2 and 6 volts across its leads.

Now the intersection of the cathode resistor and plate resistor lines *should* give us the idle current and the plate voltage — but it doesn't. We face the same task we did before of having to shift the plate curves to the right to compensate for the voltage lost across the cathode resistor. But for most situations, the intersection is close enough.

Optimal Plate Resistor Values

We know how to determine the values of the plate resistor and the cathode resistor, but which values are best? This question leads to another question: Best for what? Do we need the greatest amount of gain or the greatest voltage swing? Producing the greatest gain requires using the largest plate resistor possible and bypassing the cathode resistor with a large valued capacitor, as the following formula reveals:

$$\text{Gain} = \text{mu}R_a \,/\, (r_p + R_a)$$

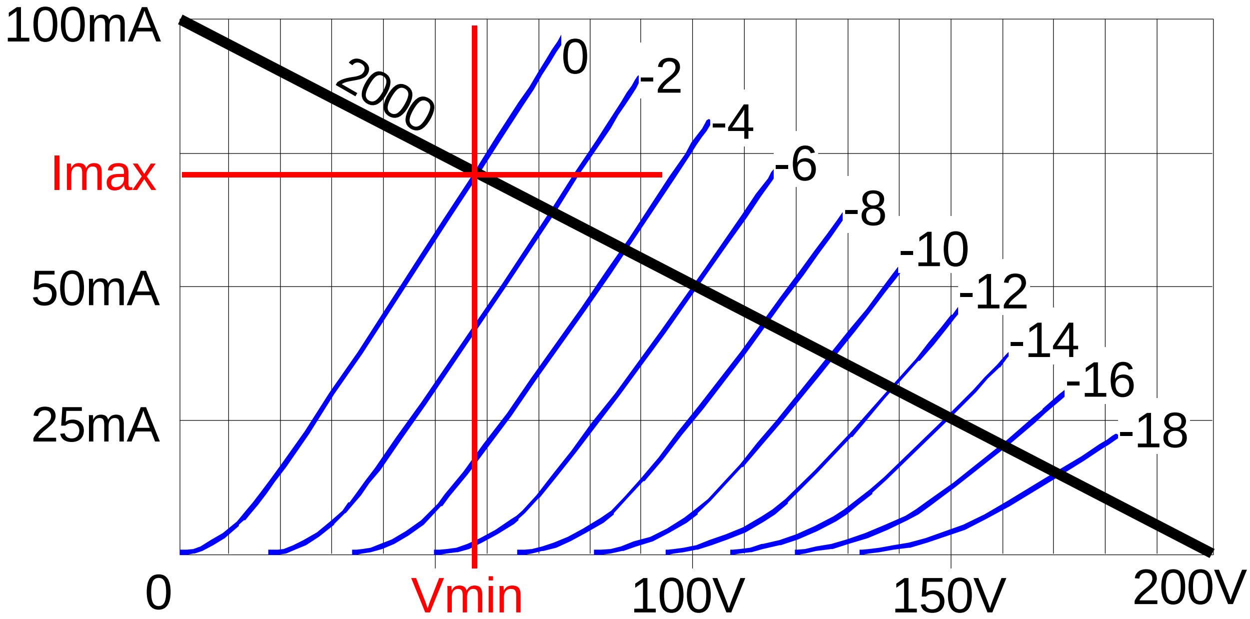
for bypassed cathode resistors and

$$\text{Gain} = \frac{\text{mu}R_a}{r_p + R_a + (\text{mu} + 1)R_k}$$

for unbypassed cathode resistors. There are limits, however. Unless we are willing to use an extremely high power supply—say, 1 kV—we must accept a mere trickle current to accommodate a plate resistor that equaled $20R_k$.

The danger of such a light idle current lies in the dragging down of the high frequency response due to the circuit's inability to charge the load (and stray) capacitance. Besides, the triode is least linear at the bottom of its conduction and the greatest amount of gain is not the same as the greatest symmetrical voltage swing.

(The hidden advantage to trickle currents is that the power supply design is greatly relived by wimpy idle currents, as the power transformer and DC filtering capacitor can be made smaller, i.e. cheaper; furthermore, it greatly extends tube life. But light current is not compatible with linearity and, over time, can lead to problems like the dreaded sleeping sickness. So, as many will agree, it is better to burn-out than rust away.)

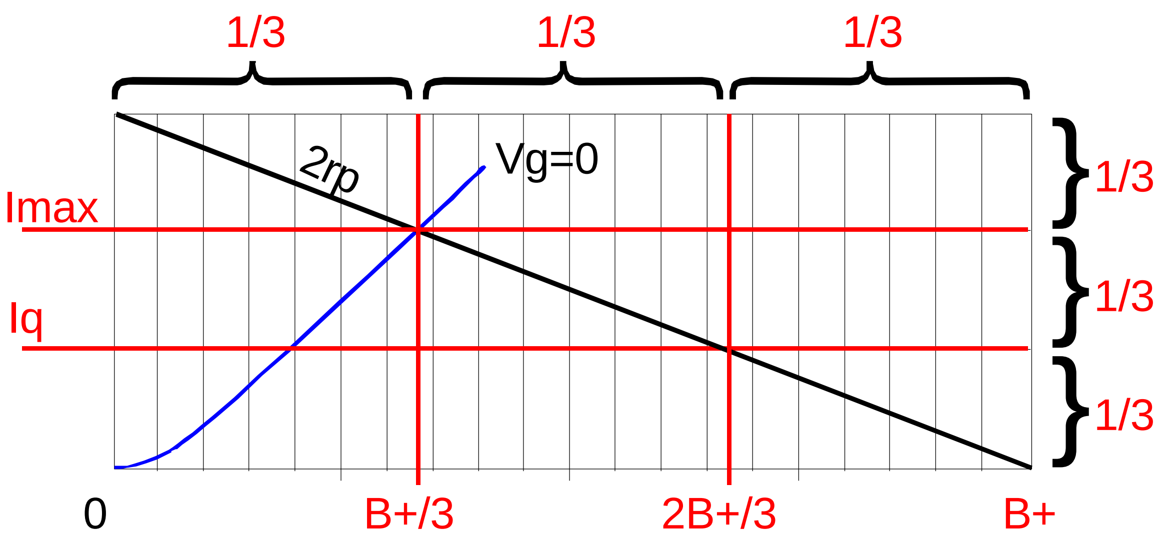


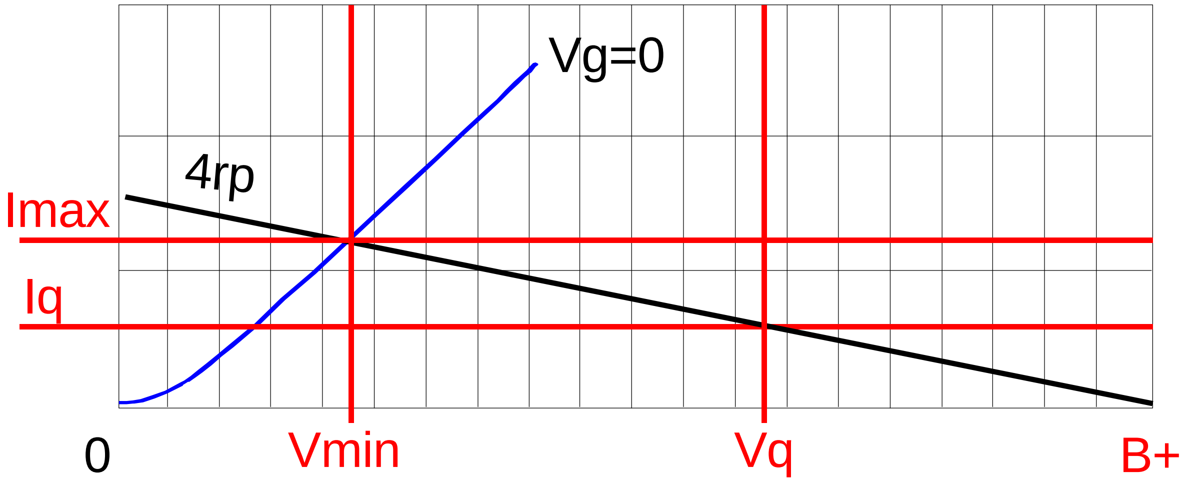
Magic Thirds and 2rp

If we specify that the plate resistor, R_a, equals two times the plate resistance and set the idle current to $V_b/6r_p$, some very sweet results obtain. How?

We must first understand some of the limitations the triode imposes. The first is that the triode's grid ceases to be high impedance once the grid becomes positive relative to the cathode, as it then becomes a diode's anode that can conduct current. So if we retain the 0-volt gridline as a boundary, then certain relationships develop. (Fixed bias will be used in this example to make the concepts clearer.)

For example, the intersection of the plate resistor loadline with the 0-volt gridline defines the maximum current draw that both triode and load resistor will experience. And, obviously, the least amount of current these elements can draw is 0. So by making $R_l = 2r_p$, we now know that the peak voltage swing positively and negatively will equal B+/3; we also know that the maximum current swing in either direction will equal the idle current, that the cathode-top voltage equals 2/3B+, and that the gain will equal 2/3 of the mu). This 1:2 ratio cleanly divides the operating rages of current and voltage swings into thirds, as shown in the graph below.





A lower valued plate resistor value will give smaller voltage swings, but a higher current swing; a higher plate resistor value, lower current swings, but a wider voltage swings. In the end, do all plate loads equal in results? No. If you do the math, you will see that $R_a=2r_p$ ratio gives the greatest power delivery into the plate load resistor (which is why this ratio is used so often in the design of SE amplifiers).

What if we do not wish to abide to $R_a=2r_p$ ratio? What if we want to use a plate resistor = $4r_p$?

To get the largest, symmetrical voltage swings with a plate resistor of our own arbitrary choosing, we must pick the optimal idle current for a given plate resistor value. Once again we need to find the range that defines the usable area of operation. With this new information, setting the optimal idle current becomes trivial. First we must find Imax, the maximum current draw the amplifier can under go without leading to positive grid current:

$$I_{max} = V_b / (R_a + r_p)$$

where V_b is the B+ voltage. Next we need to set the idle current:

$$I_q = I_{max} / 2$$

These two formulas ensure the widest possible voltage swing with *any* plate resistor value, but not the greatest power delivery into the load resistance, for that, the famous "$2r_p$" formula reenters the picture.

Optimal Cathode Resistor Values

The previous formulas were based on using grid bias to set the idle current. Adding a cathode bias resistor complicates matters, as the voltage across the cathode resistor must first be subtracted from the B+ voltage before using the above formulas, but we don't know the cathode resistor's value will be until we know the idle current, which we need the voltage drop across the cathode resistor to determine. If you think this circular, you are right. Fortunately, If the cathode resistor is bypassed, the following formula yields the approximate value for the cathode resistor:

$$R_k = (R_a + r_p) / (mu + 1)$$

When the cathode resistor is not bypassed, the following formula yields the greatest symmetrical voltage swing:

$$R_k = (R_a + r_p) / (mu - 1)$$

These formulas set the cathode resistor's value to yield the idle current that delivers the greatest symmetrical plate voltage swing by splitting the maximum current draw in half.

If you are bothered by there being two formulas, when the same valued cathode resistor is used, with the same voltage drop across it, considered this: the voltage drop is the same only at idle. When the cathode resistor is bypassed, the voltage drop is fixed for all audio frequencies, but when un-bypassed, the cathode resistor's voltage drop traces the input voltage, so that at one extreme, its voltage drop will equal 0 volts and at the other, twice the idle voltage drop.

Output Impedance

Determining the grounded cathode amplifier's output impedance is easy enough, when grid bias or a bypassed cathode resistor are used, as the resistor and the triode's r_p in parallel gives the output impedance:

$$Z_o = r_p \,||\, R_a$$

With an un-bypassed cathode resistor, the formula becomes longer:

$$Z_o = (r_p + [mu + 1]R_k) \,||\, R_a$$

PSRR

PSRR stand for "Power Supply Rejection Ratio" and it is usually expressed in decibels, dBs. As you might expect, there are two sets of formulas once again. When grid bias or a bypassed cathode resistor are used:

$$PSRR = 20Log[r_p/(r_p + R_a)]$$

With an un-bypassed cathode resistor, the formula once again becomes longer:

$$PSRR = 20Log\left(\frac{r_p + (mu + 1)R_k}{r_p + (mu + 1)R_k + R_a}\right)$$

Conclusion

A lot of material has been covered here and much of difficult to grasp at first, but after a little time, you will find that ground-cathode amplifier is really as simple as it looks. The best path to follow is to experiment with real tubes or with a computer program (Tube CAD or SPICE, for example) and compare the results with the theory, as knowing when the theory begins to break down is where art takes over from science

//JRB

